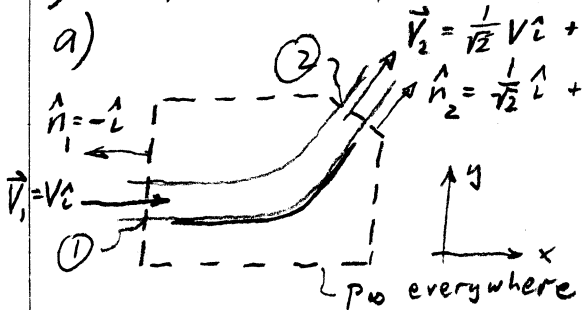
Fluids Quiz 2 Solution

Fall '07

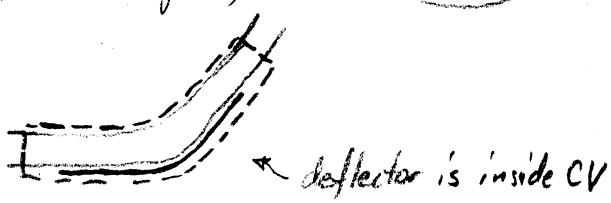
10 pts

1) Simplest CV for this problem:

a)



Equivalent CV: (same integrals)



b) $\oint \rho \vec{V}\cdot\hat{n}\, dA = \int_{①} + \int_{②}$ no contributions elsewhere, since $\vec{V}=0$ there

$\int_{①} = \rho(-V)A$

$+ \int_{②} = \rho\left(\frac{1}{\sqrt{2}}V\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}V\frac{1}{\sqrt{2}}\right)A = \rho VA$

10 pts

$$\boxed{\oint \rho \vec{V}\cdot\hat{n}\, dA = \rho(-V)A + \rho VA = 0}$$

c) $\oint \rho(\vec{V}\cdot\hat{n})\vec{V}\, dA = \int_{①} + \int_{②}$ no contributions elsewhere

$\int_{①} = \rho(-V)A\left(V\hat{i} + 0\hat{j}\right)$

$+ \int_{②} = \rho VA\left(\frac{1}{\sqrt{2}}V\hat{i} + \frac{1}{\sqrt{2}}V\hat{j}\right)$

20 pts

$$\boxed{\oint \rho(\vec{V}\cdot\hat{n})\vec{V}\, dA = -\rho V^2 A\hat{i} + \rho V^2 A\frac{1}{\sqrt{2}}\hat{i} + \rho V^2 A\frac{1}{\sqrt{2}}\hat{j} = \rho V^2 A\left[\left(\frac{1}{\sqrt{2}}-1\right)\hat{i} + \frac{1}{\sqrt{2}}\hat{j}\right]}$$

d) $\boxed{\oint p\hat{n}\, dA = \oint p_\infty \hat{n}\, dA = \vec{0}}$ using identity $\oint \phi\hat{n}\, dA = 0$

10 pts

e) $\oint \rho(\vec{V}\cdot\hat{n})\vec{V}\, dA + \oint p\hat{n}\, dA + \vec{R} = 0$ ($\oint p\hat{n}\, dA = 0$)

$$\boxed{\vec{R} = -\oint \rho(\vec{V}\cdot\hat{n})\vec{V}\, dA = \rho V^2 A\left[\left(1-\frac{1}{\sqrt{2}}\right)\hat{i} - \frac{1}{\sqrt{2}}\hat{j}\right] = \rho V^2 A\left[0.293\hat{i} - 0.707\hat{j}\right]}$$

15 pts